

# DM865 — Heuristics and Approximation Algorithms

Teachers: Marco Chiarandini & Lene Favrholt

Credits: 10 ECTS

Prerequisites:

- ✓ Programming (DM502, DM503, DM550)
- ✓ Algorithms and Datastructures (DM507)
- ✓ Complexity and Computability (DM508, DM553)
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Language: English and/or Danish

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Material: Slides + textbook + articles + starting code

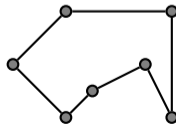
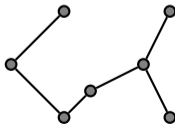
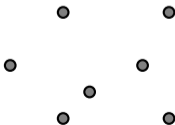
Assessment: Practical projects + oral exam

# Approximation Algorithms

A 2-approximation algorithm for Metric TSP: Double-Tree

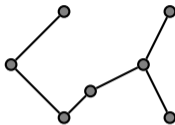
Double tree algorithm:

1.  $T \leftarrow \text{MST}$
2. Double all edges in  $T$
3.  $E_{\text{tour}} \leftarrow \text{Euler tour}$
4.  $\text{Tour} \leftarrow$  vertices in order of appearance in  $E_{\text{tour}}$



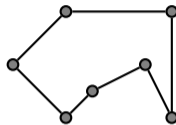
# Approximation Algorithms

A 2-approximation algorithm for Metric TSP: Double-Tree



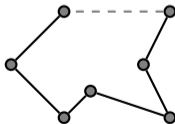
MST

$$c(\text{MST}) \leq c(\text{OPT}) :$$



Tour

$$c(\text{Tour}) \leq 2 \cdot c(\text{MST})$$



OPT  $\rightarrow$  ST

Double tree algorithm:

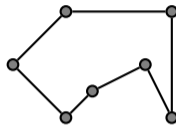
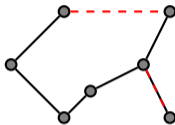
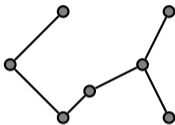
1.  $T \leftarrow \text{MST}$
2. Double all edges in  $T$
3.  $E_{\text{tour}} \leftarrow \text{Euler tour}$
4.  $\text{Tour} \leftarrow$  vertices in order of appearance in  $E_{\text{tour}}$

Christofide's algorithm:

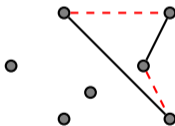
1.  $T \leftarrow \text{MST}$
2. Add minimum perfect matching of odd degree vertices in  $T$
3.  $E_{\text{tour}} \leftarrow \text{Euler tour}$
4.  $\text{Tour} \leftarrow$  vertices in order of appearance in  $E_{\text{tour}}$

# Approximation Algorithms

A  $3/2$ -approximation algorithm for Metric TSP: Christofides



$$c(M) \leq \frac{1}{2} \text{OPT} :$$



$$c(M) \leq \frac{1}{2} c(\text{Tour}) \leq c(\text{Tour}) \leq c(\text{OPT})$$

# Approximation Algorithms

## Lower bounds

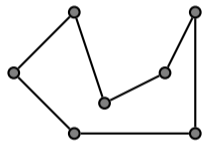
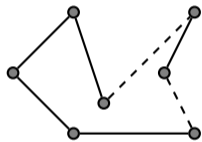
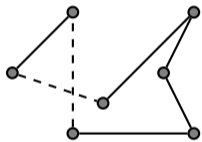
### Theorem

*For  $\alpha < \frac{185}{184}$ , there is no  $\alpha$ -approximation algorithm for Metric TSP.*

### Theorem

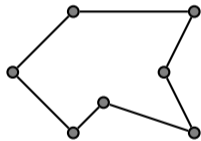
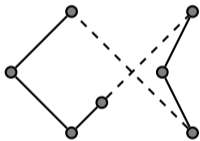
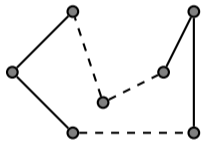
*There is no approximation algorithm for general TSP.*

# Local Search



# Metaheuristics

Trying different changes / accepting worsening changes:





# Contents

	Approx Algorithms	Local Search + Metaheuristics
Set Cover	✓	
Satisfiability	✓	✓
Traveling Salesman	✓	✓
Scheduling	✓	✓
Knapsack	✓	
Bin packing	✓	

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